Discourse That Promotes Conceptual Understanding

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As mathematics teachers, we want students to understand mathematics, not just to recite facts and execute computational procedures. We also know that allowing students to explore and have fun with mathematics may not necessarily stimulate deep thinking and promote greater conceptual understanding. Tasks that are aligned with the NCTM's curriculum standards (NCTM 1989) and that are connected to students' lives still may not challenge students to build more sophisticated understandings of mathematics. The actions of the teacher play a crucial role.

This article presents highlights from a study that demonstrates what it means to "press" students to think conceptually about mathematics (Kazemi and Stipek 1997), that is, to require reasoning that justifies procedures rather than statements of the procedures themselves. This study assessed the extent to which twenty-three upper elementary teachers supported learning and understanding during whole-class and small-group discussions. "Press for learning" was measured by the degree to which teachers (1) emphasized students' effort, (2) focused on learning and understanding, (3) supported students' autonomy, and (4) emphasized reasoning more than producing correct answers. Quantitative analyses indicated that the higher the press in the classroom, the more the students learned.

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Like researchers in other studies (e.g., Fennema et al. [1996]), we observed that when teachers helped students build on their thinking, student achievement in problem solving and conceptual understanding increased. To understand what press for learning looks like in classrooms, we studied in depth two classes with higher scores for press and two classes with lower scores, and we looked closely at mathematical activity and discourse in the classes. The high-press classroom of Ms. Carter is contrasted with the low-press classroom of Ms. Andrew.

Students in Ms. Carter's and Ms. Andrew's classes were exploring the concept of equivalence and the addition of fractions. They worked on fair-share problems, such as the following:

> I invited 8 people to a party (including me), and I had 12 brownies. How much did each person get if everyone got a fair share? Later my mother got home with 9 more brownies. We can always eat more brownies, so we shared these out equally too. This time how much brownie did each person get? How much brownie did each person eat altogether? (Corwin, Russell, and Tierney 1990, 76)

Similarities between Classrooms: Social Norms

In both Ms. Carter's and Ms. Andrew's classes, we saw students huddled in groups, materials scattered about them, figuring out how to share a batch of brownies equally among a group of people. The students seemed to be engaged in and enjoying their work. Often each group found a slightly different strategy to solve the problem. After moving from group to group, listening to and joining student conversations, both teachers stopped group activity to ask students to share their work and explain how they solved the problem.

The NCTM *Standards* document supports the view that social norms—practices such as explaining thinking, sharing strategies, and collaborating that we see in both classrooms—afford opportunities for students to engage in conceptual thinking. Many teachers establish those social norms in their classrooms quite readily. But social norms alone may not advance students' conceptual thinking.

Differences between Classrooms: Sociomathematical Norms

Although Ms. Andrew and Ms. Carter both valued problem solving and established the same social norms in their classrooms, important differences were seen in the quality of their students' engagement with the mathematics. To understand those differences, we looked more closely at the norms that guide the quality of mathematical discourse, the *sociomathematical* norms (Yackel and Cobb 1996). Teachers and students actively negotiate the sociomathematical norms that develop in any classroom. Sociomathematical norms identify what kind of talk is valued in the classroom, what counts as a mathematical explanation, and what counts as a mathematically different strategy. In the brownie problem, for example, students grapple with ideas of equivalence, part-whole relations, and the addition of fractional parts. Sociomathematical norms help us understand the ways in which fraction concepts are supported within the context of sharing and explaining strategies.

Through our study of the four classrooms, we identified four sociomathematical norms that guided students' mathematical activity and helped create a high press for conceptual thinking:

- Explanations consisted of mathematical arguments, not simply procedural summaries of the steps taken to solve the problem.
- Errors offered opportunities to reconceptualize a problem and explore contradictions and alternative strategies.
- Mathematical thinking involved understanding relations among multiple strategies.
- Collaborative work involved individual accountability and reaching consensus through mathematical argumentation.

Other norms may also contribute to a high press, but these norms captured the major differences in the way that mathematics was treated by the high- and low-press teachers.

Explaining strategies

The following examples illustrate some of the differences in the two classrooms. First, in Ms. Carter's class, explanations were not limited to descriptions of steps taken to solve a problem. They were always linked to mathematical reasons. In the following example, Ms. Carter asked Sarah and Jasmine to describe their actions and to *explain why* they chose particular partitioning strategies.

Sarah: The first four we cut them in half. [Jasmine divides squares in half on an overhead transparency. See **fig. 1.**]

Ms. Carter: Now as you explain, could you explain why you did it in half?

Sarah: Because when you put it in half, it becomes four . . . four . . . eight halves.

Ms. Carter: Eight halves. What does that mean if there are eight halves?

Sarah: Then each person gets a half.

Ms. Carter: Okay, that each person gets a half. [Jasmine labels halves 1 through 8 for each of the eight people.]

Sarah: Then there were five boxes [brownies] left. We put them in eighths.

Ms. Carter: Okay, so they divided them into eighths. Could you tell us why you chose eighths?

Sarah: It's easiest. Because then everyone will get . . . each person will get a half and [addresses Jasmine] "How many eighths?"

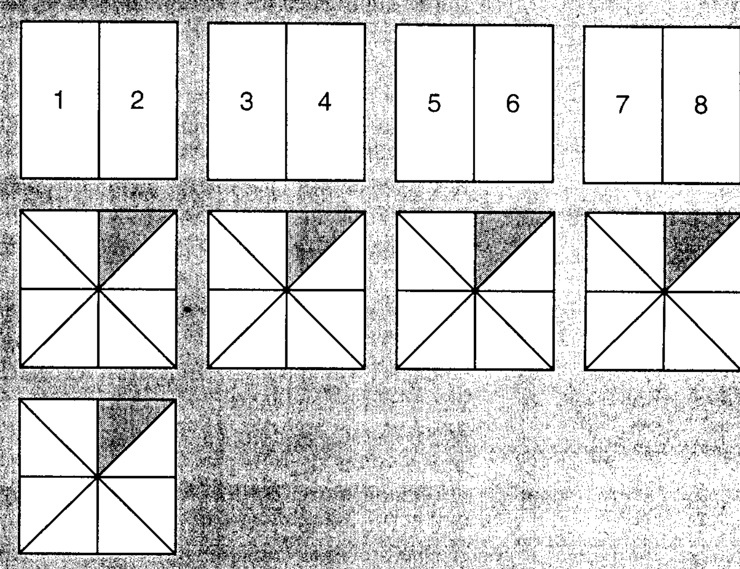
Jasmine: [Quietly] Five-eighths.

Ms. Carter: I didn't know why you did it in eighths. That's the reason. I just wanted to know why you chose eighths.

Jasmine: We did eighths because then if we did

FIGURE 1

Sharing nine brownies among eight people



eighths, each person would get each eighth, I mean one-eighth out of each brownie.

Ms. Carter: Okay, one-eighth out of each brownie. Can you just, you don't have to number, but just show us what you mean by that? I heard the words, but . . . .

[Jasmine shades in one-eighth of each of the five brownies that were divided into eighths.]

Jasmine: Person one would get this . . . [points to one-eighth].

Ms. Carter: Oh, out of each brownie.

Sarah: Out of each brownie, one person will get one-eighth.

Ms. Carter: One-eighth. Okay. So how much did they get if they got their fair share?

Jasmine and Sarah: They got a half and five-eighths.

Ms. Carter: Do you want to write that down at the top, so I can see what you did?

[Jasmine writes 1/2 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 at the top of the overhead transparency.]

The exchange among Sarah, Jasmine, and Ms. Carter highlighted the conceptual focus of the lesson on fair share. Ms. Carter asked Sarah to explain the importance of having eight halves and why the partitioning strategy using eighths made sense. After Jasmine gave a verbal justification, Ms. Carter continued to press her to link her verbal response to the appropriate pictorial representation—by shading the pieces—and to the symbolic representation—by writing the sum of the fractions.

The same degree of press did not exist in Ms. Andrew's classroom. Ms. Andrew's students engaged in the same social practice of sharing their strategies with the class, but the mathematical content of classroom conversations was different. Students shared solutions by giving procedural summaries of the steps they took to solve the problem, as demonstrated by the following exchange, in which Raymond described his solution for sharing twelve brownies among eight people. Ms. Andrew had drawn twelve squares on the chalkboard.

[Raymond divides four of the brownies in half.]

Ms. Andrew: Okay, now would you like to explain to us what . . . loud . . . .

Raymond: Each one gets one, and I give them a half.

Ms. Andrew: So each person got how much?

Raymond: One and one-half.

Ms. Andrew: One-half?

Raymond: No, one and one-half.

Ms. Andrew: So you're saying that each one gets one and one-half. Does that make sense? [After a chorus of "yeahs" comes from students, Ms. Andrew moves on to another problem.]

Unlike Ms. Carter, Ms. Andrew did not ask her students to justify why they chose a particular partitioning strategy. Instead, Ms. Andrew often asked questions that required a show of hands or yes-no responses, such as "How many people agree?" "Does this make sense?" or "Do you think that was a good answer?" Ms. Andrew wanted to engage her students in the activity and to see if they understood, but the questions she asked yielded general responses without revealing specific information about the students' thinking.

Reacting to mathematical errors

By emphasizing mathematical reasons for actions, Ms. Carter created opportunities for her students to prove that their solutions were correct. She resisted telling students that an answer or reason was wrong, and she invited others to respond to incorrect solutions. Ms. Carter modeled the kinds of questions that may help students think through their own confusion by using their existing knowledge. Those questions usually involved graphical representations of the fractions. In small groups, students challenged one another when they disagreed on a solution and helped one another find errors.

The interaction among Ms. Carter, Jasmine, and Sarah continued with the following conversation.

[Jasmine writes 1/2 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 at the top of the overhead transparency.]

Ms. Carter: Okay, so that's what you did. So how much was that in all?

Jasmine: It equals 1 1/8 or 6/8.

Ms. Carter: So she says it can equal 6 and 6/8? [She misheard Jasmine.]

Jasmine: No, it can equal 6/8 or it can equal 1 1/8.

Ms. Carter: Okay, so you have two different answers. Could you write them down so people can see it? And boys and girls, I'd like you to respond to what they've written up here. She says it either could equal 6/8 or 1 1/8.

Ms. Carter: Matthew had his hand up and was thinking about it. Someone from team 5. Anybody from team 6 that has a response? Right now. I'm just going to let you look a minute. See if anyone has a response. Andrew, you had your hand up, is that right? [waits] Still only have four hands up. I wonder if you're all looking up here and seeing? She's given us two answers here, 6/8 or it can equal 1 1/8. Okay, could those four people right now . . . do you agree with both answers?

Students: No . . . .

Ms. Carter: Do you have a reason why you don't agree? Don't explain it to me, but do you have a reason? Raise your hand if you have a reason why you don't agree. [hands] One, two, three, four, five, six . . . okay. Would those six people please stand? Okay. Would you please, you're going to be in charge of explaining why you don't agree to your team. [She assigns those six students to teams.] Right now, if you don't agree, would you please tell them what you think the answer is and why you don't agree. Go ahead. Explain.

Ms. Carter could have stepped in and pointed out why 6/8 and 1 1/8 are not equal. Instead, her response to this mistake was to encourage her students to explore the error by providing the conceptual reasons for why 6/8 and 1 1/8 are not equal. She engaged the entire class in thinking about which solution was correct instead of talking with only the two presenters or correcting their mistake herself, and she created an opportunity for her students to practice articulating their thinking.

The mistake also created an opportunity for the entire class to explore contradictions in the solution and to build an understanding of fractional equivalence and the addition of fractions by using an area model. This type of activity and discourse was typical in Ms. Carter's classroom. In a whole-class discussion, each group shared its proof that 1 1/8 was correct. Neither the students nor Ms. Carter belittled, penalized, or discredited anyone who made a mistake. The atmosphere of mutual respect between the students and Ms. Carter allowed the class to think about and build conceptual understandings eagerly.

Ms. Andrew treated errors differently. Note how she provides the mathematical reasoning when three boys explained their solution for sharing five brownies among six people.

Ms. Andrew: They got 1/2, you already said that. And then 1/6 and then another sixth. So, how many sixths did they get?

Anthony: One, two.

Ryan: One, two.

Joe: 1/12.

Ms. Andrew: What did you say? [to Joe] They got two . . .

Ryan: Sixths.

Anthony: 2/12.

Joe: 2/6.

Ms. Andrew: 2/6 [confirming the right answer]. Why did you say 2/12? Because there are twelve parts altogether?

Anthony: Yeah.

Ms. Andrew: Okay, be sure not to get confused. Because there are two brownies not one. Perfect. Good, good job.

At first, the boys appeared to be guessing the answer to Ms. Andrew's question. She focused on Joe once he stated the right answer. Although she predicted accurately why Anthony said 2/12, she did not ask him to think about why his answer did not work. Instead, she asked and answered the question herself and did not press Anthony to sort out his confusion. Her statement "Because there are two brownies, not one" was left unexplained. As this example illustrates, limited opportunity was available for the members of the group to engage in conceptual thinking about what 1/6 and 1/12 signify and how the graphical representation is linked to the numeric representation.

Both Ms. Carter and Ms. Andrew allowed students to make mistakes. That social norm, however, was not enough to press students to examine their work conceptually. Both teachers wanted their students to learn from their mistakes, but Ms. Andrew often supplied the conceptual thinking for her students. In Ms. Carter's class, inadequate solutions served as entry points for further mathematical discussion.

Comparing strategies

Students in both classrooms worked together, shared their strategies, and were praised for their efforts. Students in both classrooms attended to nonmathematical similarities between shared solutions, such as the layout of the paper or the uses of color. In Ms. Andrew's class, strategies were typically offered one after the other, with discussion limited to nonmathematical aspects of students' work. For example, a pair of students noted that they cut the paper brownies and pasted the pieces under stick-figure illustrations. Another pair had drawn lines from the fractional parts of the brownies to the individuals that received them. Although the partitioning strategy in both was the same, students viewed the strategies as different because the representations were different. Ms. Carter, however, pressed her students to go beyond their initial observations and reflect on the mathematical similarities and differences between strategies.

Accountability and consensus

In inquiry-based classrooms, students often work together to share interpretations and solutions and construct new understandings. Important differences arose between Ms. Andrew's and Ms. Carter's classes in the way in which they emphasized individual accountability and consensus. Ms. Carter required her students to make sure that each person contributed to, and understood the mathematics involved in, the group's solution. If students disagreed about an answer, she encouraged them to prove their answers mathematically and to work until they arrived at a consensus. If she noticed that students were not listening to others during an activity, she reminded them that they had to prove their solutions and that each group member must be prepared to discuss the reasons for the solution in front of the class. As a result, the distribution of work was more equitable. Students listened to one another's ideas and evaluated their appropriateness before using them.

Ms. Andrew did not describe and discuss collaboration beyond the general directive to "work with a partner" or "remember to work together." Neither individual accountability nor consensus emerged as topics of discussion in whole-class activity. Typically, only one person would be in control of group work at any particular time and would complete most of the work.

Conclusion

We saw a consistently higher press for conceptual thinking in Ms. Carter's class. She took her students' ideas seriously as they engaged in building mathematical concepts. In both whole-class discussions and small-group work, all students were accountable for participating in an intellectual climate characterized by argument and justification. Four sociomathematical norms governed mathematical discourse in Ms. Carter's classroom: explanations were supported by mathematical reasons, mistakes created opportunities to engage further with mathematical ideas, students drew mathematical connections between strategies, and each student was accountable for the work of the group.

When teachers create a high press for conceptual thinking, mathematics drives not only the activities but the students' explanations as well. As a result, student achievement in problem solving and conceptual understanding increases.

Action Research Ideas

- Over time, listen for differences in the number of times that you that interrupt a student's explanation, restate a student's explanation, or provide a solution strategy. By keeping a daily log, notice any changes in the nature and quantity of your responses.
- *(a)* Identify the social norms and the sociomathematical norms that characterize your classroom. *(b)* Discuss the issue of sociomathematical norms with a colleague. Share your goals and the problems that you expect to encounter. Continue to discuss your progress with your colleague over time. Encourage your colleague to engage in a similar program to create a higher press. *(c)* Observe and discuss each other's teaching.
- *(a)* Reflect on the discourse associated with a problem recently discussed in your classroom. Using a four-point scale from 0 (low press) to 4 (high press), rate the discourse according to each of the sociomathematical norms that characterize Ms. Carter's classroom. *(b)* Set personal goals for each of the sociomathematical norms. Use such questions as the following to help establish a high press: "How can you prove that your answer is right? Can you prove it in more than one way? How is your strategy mathematically different from, or mathematically like, that of [another student]? Do you agree or disagree with [another student's] solution? Why? Why does [strategy *x*] work? Why does [strategy *y*] not work?" *(c)* After four weeks, reevaluate your classroom, using the same scale and the same sociomathematical norms. Note your areas of improvement, and set new goals for the next four weeks.

References

Corwin, R. B., S. J. Russell, and C. C. Tierney. *Seeing Fractions: Representations of Wholes and Parts. A Unit for the Upper Elementary Grades.* Sacramento, Calif.: Technical Education Research Center, California Department of Education, 1990.

Fennema, Elizabeth, Thomas P. Carpenter, Megan L. Franke, Linda Levi, Victoria R. Jacobs, and Susan B. Empson. "A Longitudinal Study of Learning to Use Children's Thinking in Mathematics Instruction." *Journal for Research in Mathematics Education* 27 (July 1996): 403–34.

Kazemi, E., and D. Stipek. "Pressing Students to Be Thoughtful: Promoting Conceptual Thinking in Mathematics." Paper presented at the annual meeting of the American Educational Research Association, Chicago, 1997.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

Yackel, Erna, and Paul Cobb. "Sociomathematical Norms, Argumentation, and Autonomy in Mathematics." *Journal for Research in Mathematics Education* 27 (July, 1996): 458–77.

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